Physics Letters B 666 (2008) 477–482

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Analysis of the $B \to a_1(1260)$ form-factors with light-cone QCD sum rules

Zhi-Gang Wang

Department of Physics, North China Electric Power University, Baoding 071003, PR China

ARTICLE INFO

Article history:
Received 15 April 2008
Received in revised form 11 August 2008
Accepted 11 August 2008
Available online 13 August 2008
Editor: B. Grinstein

PACS:
12.38.Lg
13.20.He

Keywords:
B meson
Light-cone QCD sum rules

ABSTRACT

In this article, we calculate the $B \to a_1(1260)$ form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$ and $A(q^2)$ with the B-meson light-cone QCD sum rules. Those form-factors are basic parameters in studying the exclusive non-leptonic two-body decays $B \to AP$ and semi-leptonic decays $B \to Al\nu_l$, $B \to Al\bar{l}$. Our numerical results are consistent with the values from the (light-cone) QCD sum rules. The main uncertainty comes from the parameter ω_0 (or λ_B), which determines the shapes of the two-particle and three-particle light-cone distribution amplitudes of the B-meson, it is of great importance to refine this parameter.

1. Introduction

The weak $B \to P, V, A$ form-factors with $P = \pi, K$, $V = \rho, K^*$ and $A = a_1, K_1$ final states are basic input parameters in studying the exclusive semi-leptonic decays $B \to P(V, A)l\nu_l$, $B \to P(V, A)\bar{l}l$ and radiative decays $B \to V(A)\gamma$, they also determine the factorizable amplitudes in the non-leptonic charmless two-body decays $B \to PP(AP, PV, VV)$. Those decays can be used to determine the CKM matrix elements and to test the Standard Model, however, it is a great challenge to pin down the uncertainties of the form-factors to obtain more precise results. The exclusive semi-leptonic decays $B \to P(V)l\nu_l$, $B \to P(V)\bar{l}l$ and radiative decays $B \to V\gamma$ and hadronic two-body decays $B \to PP(PV, VV)$ have been studied extensively [1–7], while the decays $B \to AP, VA$ have been calculated with the QCD factorization approach [8–10], generalized factorization approach [11,12], etc. It is more easy to deal with the exclusive semi-leptonic precesses than the non-leptonic precesses, and there have been many works on the relevant form-factors $B \to \pi$, $B \to \rho$ in determining the CKM matrix element V_{ub} [13–16]. The $B \to a_1(1260)$ form-factors have been studied with the covariant light-front approach [17], ISGW2 quark model [18], quark–meson model [19], QCD sum rules [20], light-cone QCD sum rules [9] and perturbative QCD [21]. However, the values from different theoretical approaches differ greatly from each other.

The BaBar Collaboration and Belle Collaboration have measured the charmless hadronic decays $B^0 \to a_1^{\pm}\pi^{\mp}$ [22,23]. Moreover, the BaBar Collaboration has measured the time-dependent CP asymmetries in the decays $B^0 \to a_1^{\pm}\pi^{\mp}$ with $a_1^{\mp} \to \pi^{\mp}\pi^{\pm}\pi^{\mp}$, from the measured CP parameters, we can determine the decay rates of $a_1^{+}\pi^{-}$ and $a_1^{-}\pi^{+}$ respectively [24]. Recently, the BaBar Collaboration has reported the observation of the decays $B^{\pm} \to a_1^0\pi^{\pm}, a_1^{\pm}\pi^0$, $B^+ \to a_1^+K^0$ and $B^0 \to a_1^-K^+$ [25,26]. So it is interesting to re-analyze the $B \to a_1$ form-factors with the B-meson light-cone QCD sum rules [27].

In Ref. [27], the authors obtain new sum rules for the $B \to \pi, K, \rho, K^*$ form-factors from the correlation functions expanded near the light-cone in terms of the B-meson distribution amplitudes, and suggest QCD sum rules motivated models for the three-particle B-meson light-cone distribution amplitudes, which satisfy the relations given in Ref. [28]. In Ref. [28], the authors derive exact relations between the two-particle and three-particle B-meson light-cone distribution amplitudes from the QCD equations of motion and heavy-quark symmetry. The two-particle B-meson light-cone distribution amplitudes have been studied with the QCD sum rules and renormalization group equation [29–35]. Although the QCD sum rules cannot be used for a direct calculation of the distribution amplitudes, it can provide constraints which have to be implemented within the QCD motivated models (or parameterizations) [32].

E-mail address: wangzgyiti@yahoo.com.cn.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.08.014

The B-meson light-cone distribution amplitudes play an important role in the exclusive B-decays, the inverse moment of the two-particle light-cone distribution amplitude $\phi_+(\omega)$ enters many factorization formulas (for example, see Refs. [3,4]). However, the light-cone distribution amplitudes of the B-meson are received relatively little attention comparing with the ones of the light pseudoscalar mesons and vector mesons, our knowledge about the nonperturbative parameters which determine those light-cone distribution amplitudes is limited and an additional application (or estimation) based on QCD is useful.

In this Letter, we use the B-meson light-cone QCD sum rules to study the $B \to a_1$ form-factors. The semi-leptonic decays $B \to Al\nu_l$ can be observed at the LHCb, where the $b\bar{b}$ pairs will be copiously produced with the cross section about 500 μb.

We can also study the form-factors with the light-cone QCD sum rules using the light-cone distribution amplitudes of the axial-vector mesons. Recently, the twist-2 and twist-3 light-cone distribution amplitudes of the axial-vector mesons have been calculated with the QCD sum rules [36].

The B-meson light-cone QCD sum rules have given reasonable values for the $B \to \pi, K, \rho, K^*$ form-factors [27], so it is interesting to study the $B \to a_1$ form-factors and cross-check the properties of the B-meson light-cone distribution amplitudes. Furthermore, it is necessary to investigate the form-factors with different approaches and compare the predictions of different approaches.

The Letter is arranged in the following way. In Section 2, we derive the $B \to a_1(1260)$ form-factors with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and Section 4 is reserved for conclusion.

2. $B \to a_1(1260)$ form-factors with light-cone QCD sum rules

In the following, we write down the definitions for the weak form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$, $V_0(q^2)$ and $A(q^2)$ [17],

$$\langle a_1(p)\big|J_\mu(0)\big|B(P)\rangle = i\left\{(M_B - M_a)\epsilon^*_\mu V_1\big(q^2\big) - \frac{\epsilon^* \cdot P}{M_B - M_a}(P+p)_\mu V_2\big(q^2\big) - 2M_a\frac{\epsilon^* \cdot P}{q^2}q_\mu\big[V_3\big(q^2\big) - V_0\big(q^2\big)\big]\right\}, \tag{1}$$

$$\langle a_1(p)\big|J^A_\mu(0)\big|B(P)\rangle = \frac{1}{M_B - M_a}\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu(P+p)_\alpha q_\beta A\big(q^2\big), \tag{2}$$

where

$$V_3\big(q^2\big) = \frac{M_B - M_a}{2M_a}V_1\big(q^2\big) - \frac{M_B + M_a}{2M_a}V_2\big(q^2\big), \qquad J_\mu(x) = \bar{d}(x)\gamma_\mu b(x), \qquad J^A_\mu(x) = \bar{d}(x)\gamma_\mu\gamma_5 b(x), \tag{3}$$

$V_0(0) = V_3(0)$, and the ϵ_μ is the polarization vector of the axial-vector meson $a_1(1260)$. We study the weak form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$, $V_0(q^2)$ and $A(q^2)$ with the two-point correlation functions $\Pi^i_\mu(p,q)$,

$$\Pi^i_{\mu\nu}(p,q) = i\int d^4x\, e^{ip\cdot x}\langle 0|T\big\{J^a_\mu(x)J^i_\mu(0)\big\}|B(P)\rangle, \quad J^a_\mu(x) = \bar{u}(x)\gamma_\mu\gamma_5 d(x), \tag{4}$$

where $J^i_\mu(x) = J_\mu(x)$ and $J^A_\mu(x)$ respectively, and the axial-vector current $J^a_\mu(x)$ interpolates the axial-vector meson $a_1(1260)$. The correlation functions $\Pi^i_\mu(p,q)$ can be decomposed as

$$\begin{aligned}
&\Pi^1_\mu(p,q) = \Pi_A g_{\mu\nu} + \Pi_B q_\mu p_\nu + \Pi_C p_\mu q_\nu + \Pi_D q_\mu q_\nu + \Pi_D p_\mu p_\nu, \\
&\Pi^2_\mu(p,q) = \Pi_2\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta + \cdots
\end{aligned} \tag{5}$$

due to Lorentz covariance. In this Letter, we derive the sum rules with the tensor structures $g_{\mu\nu}$, $q_\mu p_\nu$ and $\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta$ respectively to avoid contaminations from the π meson.

According to the basic assumption of current–hadron duality in the QCD sum rules approach [37,38], we can insert a complete series of intermediate states with the same quantum numbers as the current operator $J^a_\mu(x)$ into the correlation functions $\Pi^i_\mu(p,q)$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the meson $a_1(1260)$, the correlation functions $\Pi^i_{\mu\nu}(p,q)$ can be expressed in the following form:

$$\Pi^1_{\mu\nu}(p,q) = -\frac{if_a M_a(M_B - M_a)V_1(q^2)}{M_a^2 - p^2}g_{\mu\nu} + \frac{2if_a M_a V_2(q^2)}{(M_B - M_a)(M_a^2 - p^2)}q_\mu p_\nu + \cdots, \tag{6}$$

$$\Pi^2_{\mu\nu}(p,q) = \frac{2f_a M_a A(q^2)}{(M_B - M_a)(M_a^2 - p^2)}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta + \cdots, \tag{7}$$

where we have used the standard definition for the decay constant f_a, $\langle 0|J^a_\mu(0)|a_1(p)\rangle = f_a M_a\epsilon_\mu$.

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi^i_\mu(p,q)$ in perturbative QCD theory. The calculations are performed at the large space-like momentum region $p^2 \ll 0$ and $0 \leqslant q^2 < m_b^2 + m_b p^2/\bar{\Lambda}$, where $M_B = m_b + \bar{\Lambda}$ in the heavy quark limit. We write down the propagator of a massless quark in the external gluon field in the Fock–Schwinger gauge and the light-cone distribution amplitudes of the B meson firstly [39],

$$\langle 0|T\big\{q_i(x_1)\bar{q}_j(x_2)\big\}|0\rangle = i\int\frac{d^4k}{(2\pi)^4}e^{-ik(x_1-x_2)}\left\{\frac{\not{k}}{k^2}\delta_{ij} - \int_0^1 dv\, G^{ij}_{\mu\nu}\big(vx_1 + (1-v)x_2\big)\left[\frac{1}{2}\frac{\not{k}}{k^4}\sigma^{\mu\nu} - \frac{1}{k^2}v(x_1-x_2)^\mu\gamma^\nu\right]\right\},$$

$$\langle 0|\bar{q}_\alpha(x)h_{v\beta}(0)|B(v)\rangle = -\frac{if_B m_B}{4}\int_0^\infty d\omega\, e^{-i\omega v\cdot x}\left\{(1+\not{v})\left[\phi_+(\omega) - \frac{\phi_+(\omega) - \phi_-(\omega)}{2v\cdot x}\not{x}\right]\gamma_5\right\}_{\beta\alpha},$$

$$\langle 0|\bar{q}_\alpha(x)G_{\lambda\rho}(ux)h_{v\beta}(0)|B(v)\rangle = \frac{f_B m_B}{4}\int_0^\infty d\omega \int_0^\infty d\xi\, e^{-i(\omega+u\xi)v\cdot x}\Big\{(1+\not v)\Big[(v_\lambda\gamma_\rho - v_\rho\gamma_\lambda)\big(\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)\big) - i\sigma_{\lambda\rho}\Psi_V(\omega,\xi) - \frac{x_\lambda v_\rho - x_\rho v_\lambda}{v\cdot x}X_A(\omega,\xi) + \frac{x_\lambda\gamma_\rho - x_\rho\gamma_\lambda}{v\cdot x}Y_A(\omega,\xi)\Big]\gamma_5\Big\}_{\beta\alpha}, \quad (8)$$

where

$$\phi_+(\omega) = \frac{\omega}{\omega_0^2}e^{-\frac{\omega}{\omega_0}}, \qquad \phi_-(\omega) = \frac{1}{\omega_0}e^{-\frac{\omega}{\omega_0}}, \qquad \Psi_A(\omega,\xi) = \Psi_V(\omega,\xi) = \frac{\lambda_E^2}{6\omega_0^4}\xi^2 e^{-\frac{\omega+\xi}{\omega_0}},$$
$$X_A(\omega,\xi) = \frac{\lambda_E^2}{6\omega_0^4}\xi(2\omega-\xi)e^{-\frac{\omega+\xi}{\omega_0}}, \qquad Y_A(\omega,\xi) = -\frac{\lambda_E^2}{24\omega_0^4}\xi(7\omega_0 - 13\omega + 3\xi)e^{-\frac{\omega+\xi}{\omega_0}}, \quad (9)$$

the ω_0 and λ_E^2 are some parameters of the B-meson light-cone distribution amplitudes.

Substituting the d quark propagator and the corresponding B-meson light-cone distribution amplitudes into the correlation functions $\Pi_\mu^i(p,q)$, and completing the integrals over the variables x and k, finally we obtain the representation at the level of quark–gluon degrees of freedom. In this Letter, we take the three-particle B-meson light-cone distribution amplitudes suggested in Ref. [27], they obey the powerful constraints derived in Ref. [28] and the relations between the matrix elements of the local operators and the moments of the light-cone distribution amplitudes, if the conditions $\omega_0 = \frac{2}{3}\bar{\Lambda}$ and $\lambda_E^2 = \lambda_H^2 = \frac{3}{2}\omega_0^2 = \frac{2}{3}\bar{\Lambda}^2$ are satisfied [29].

In the region of small ω, the exponential form of distribution amplitude $\phi_+(\omega)$ is numerically close to the more elaborated model (or the BIK distribution amplitude (BIK DA)) suggested in Ref. [32],

$$\phi_+(\omega, \mu = 1\ \text{GeV}) = \frac{4\omega}{\pi\lambda_B(1+\omega^2)}\left[\frac{1}{1+\omega^2} - 2\frac{\sigma_B - 1}{\pi^2}\ln\omega\right], \quad (10)$$

where $\omega_0 = \lambda_B$. The parameters λ_B and σ_B are determined from the heavy quark effective theory QCD sum rules including the radiative and nonperturbative corrections. There are other phenomenological models for the two-particle B-meson light-cone distribution amplitudes, for example, the k_T factorization formalism [40,41], in this article, we use the QCD sum rules motivated models.

After matching with the hadronic representation below the continuum threshold s_0, we obtain the following three sum rules for the weak form-factors $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$ respectively,

$$\begin{aligned} V_1(q^2) = \frac{1}{f_a M_a(M_B - M_a)}e^{\frac{M_a^2}{M^2}}\Bigg\{ &-\frac{1}{2}f_B M_B M^2\int_0^{\sigma_0} d\sigma\, \phi_+(\omega')\frac{d}{d\sigma}e^{-\frac{s}{M^2}} \\ &- \frac{f_B M_B}{2}\int_0^{\sigma_0} d\sigma\int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^\infty \frac{d\xi}{\xi}\big[\Psi_A(\omega,\xi) - \Psi_V(\omega,\xi)\big]\frac{d}{d\sigma}\frac{1}{\bar\sigma}e^{-\frac{s}{M^2}} \\ &+ \frac{f_B M_B^2}{M^2}\int_0^{\sigma_0} d\sigma\int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^\infty \frac{d\xi}{\xi}\frac{(1-2u)[3\tilde{X}_A(\omega,\xi) - 2\tilde{Y}_A(\omega,\xi)]}{\bar\sigma^2}e^{-\frac{s}{M^2}} \\ &- f_B\int_0^{\sigma_0} d\sigma\int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^\infty \frac{d\xi}{\xi}\frac{(1-2u)\tilde{X}_A(\omega,\xi)}{\bar\sigma^3}e^{-\frac{s}{M^2}}\left[\frac{\tilde{M}_B^4 - 4sM_B^2}{2M^4} - 2\frac{\tilde{M}_B^2 - 2M_B^2}{M^2} + 1\right]\Bigg\}, \end{aligned} \quad (11)$$

$$\begin{aligned} V_2(q^2) = \frac{M_B - M_a}{2f_a M_a}e^{\frac{M_a^2}{M^2}}\Bigg\{ &f_B M_B\int_0^{\sigma_0} d\sigma\left[\phi_+(\omega')\frac{1-2\sigma}{\bar\sigma} + \frac{2M_B}{M^2}\big[\tilde\phi_+(\omega') - \tilde\phi_-(\omega')\big]\frac{\sigma}{\bar\sigma}\right]e^{-\frac{s}{M^2}} \\ &+ \frac{f_B M_B}{M^2}\int_0^{\sigma_0} d\sigma\int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^\infty \frac{d\xi}{\xi}\frac{(2\sigma - 3)[\Psi_A(\omega,\xi) - \Psi_V(\omega,\xi)]}{\bar\sigma^2}e^{-\frac{s}{M^2}} \\ &+ \frac{f_B}{M^2}\int_0^{\sigma_0} d\sigma\int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^\infty \frac{d\xi}{\xi}(1-2u)\tilde{X}_A(\omega,\xi)\left(6 + \frac{d}{d\sigma}\right)\frac{1}{\bar\sigma^2}e^{-\frac{s}{M^2}} \\ &- \frac{4f_B M_B^2}{M^4}\int_0^{\sigma_0} d\sigma\int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^\infty \frac{d\xi}{\xi}(1-2u)\tilde{Y}_A(\omega,\xi)\frac{\sigma}{\bar\sigma^2}e^{-\frac{s}{M^2}} \\ &- \frac{4f_B}{M^2}\int_0^{\sigma_0} d\sigma\int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^\infty \frac{d\xi}{\xi}\frac{(1-2u)\tilde{X}_A(\omega,\xi)}{\bar\sigma^3}\left[2 - \sigma - \frac{2s - \sigma\tilde{M}_B^2}{2M^2}\right]e^{-\frac{s}{M^2}}\Bigg\}, \end{aligned} \quad (12)$$

$$A(q^2)=\frac{M_B-M_a}{2f_aM_a}e^{\frac{M_a^2}{M^2}}\Bigg\{f_BM_B\int_0^{\sigma_0}d\sigma\,\frac{\phi_+(\omega')}{\bar\sigma}e^{-\frac{s}{M^2}}+\frac{f_BM_B}{M^2}\int_0^{\sigma_0}d\sigma\int_0^{\sigma M_B}d\omega\int_{\sigma M_B-\omega}^{\infty}\frac{d\xi}{\xi}\frac{[\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)]}{\bar\sigma^2}e^{-\frac{s}{M^2}}$$
$$+\frac{f_B}{M^2}\int_0^{\sigma_0}d\sigma\int_0^{\sigma M_B}d\omega\int_{\sigma M_B-\omega}^{\infty}\frac{d\xi}{\xi}(1-2u)\tilde X_A(\omega,\xi)\frac{d}{d\sigma}\frac{1}{\bar\sigma^2}e^{-\frac{s}{M^2}}\Bigg\},\tag{13}$$

where

$$s=M_B^2\sigma-\frac{\sigma}{\bar\sigma}q^2,\qquad \omega'=\sigma M_B,\qquad \bar\sigma=1-\sigma,$$
$$\sigma_0=\frac{s_0+M_B^2-q^2-\sqrt{(s_0+M_B^2-q^2)^2-4s_0M_B^2}}{2M_B^2},\qquad u=\frac{\sigma M_B-\omega}{\xi},\qquad \tilde M_B^2=M_B^2(1+\sigma)-\frac{1}{\bar\sigma}q^2,$$
$$\tilde X_A(\omega,\xi)=\int_0^{\omega}d\lambda\,X_A(\lambda,\xi),\qquad \tilde Y_A(\omega,\xi)=\int_0^{\omega}d\lambda\,Y_A(\lambda,\xi),\qquad \tilde\phi_\pm(\omega)=\int_0^{\omega}d\lambda\,\phi_\pm(\lambda).\tag{14}$$

In Ref. [31], Lange and Neubert observe that the evolution effects drive the light-cone distribution amplitude $\phi_+(\omega)$ toward a linear growth at the origin and generate a radiative tail that falls off slower than $\frac{1}{\omega}$, even if the initial function has an arbitrarily rapid falloff, which implies the normalization integral of the $\phi_+(\omega)$ is ultraviolet divergent. In this Letter, we derive the sum rules without the radiative $\mathcal{O}(\alpha_s)$ corrections, the ultraviolet behavior of the $\phi_+(\omega)$ plays no role at the leading order ($\mathcal{O}(1)$). Furthermore, the duality thresholds in the sum rules are well below the region where the effect of the tail becomes noticeable. The nontrivial renormalization of the B-meson light-cone distribution amplitude is so far known only for the $\phi_+(\omega)$, we use the light-cone distribution amplitudes of order $\mathcal{O}(1)$, which satisfy all QCD constraints.

3. Numerical results and discussion

The input parameters are taken as $\omega_0=\lambda_B(\mu)=(0.46\pm0.11)$ GeV, $\mu=1$ GeV [32], $\lambda_E^2=(0.11\pm0.06)$ GeV^2 [29], $M_a=(1.23\pm0.06)$ GeV, $f_a=(0.238\pm0.010)$ GeV, $s_0=(2.55\pm0.15)$ GeV^2 [36], $M_B=5.279$ GeV, $f_B=(0.18\pm0.02)$ GeV [42,43].

The Borel parameters in the three sum rules are taken as $M^2=1.1$–1.5 GeV^2, in this region, the values of the weak form-factors $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$ are stable enough.

Taking into account all the uncertainties, we obtain the numerical values of the weak form-factors $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$, which are shown in Fig. 1, at zero momentum transfer,

$$V_1(0)=0.67^{+0.33}_{-0.21},\qquad V_2(0)=0.31^{+0.18}_{-0.11},\qquad V_3(0)=0.29^{+0.07}_{-0.06},\qquad V_0(0)=0.29^{+0.07}_{-0.06},\qquad A(0)=0.41^{+0.20}_{-0.13}.\tag{15}$$

The form-factors can be parameterized in the double-pole form,

$$F_i(q^2)=\frac{F_i(0)}{1+a_Fq^2/M_b^2+b_Fq^4/M_B^4},\tag{16}$$

where we use the notation $F_i(q^2)$ to denote the $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$, the a_F and b_F are the corresponding coefficients and their values are presented in Table 3.

In calculation, we observe the dominating contributions in the three sum rules come from the two-particle B-meson light-cone distribution amplitudes, the contributions from the three-particle B-meson light-cone distribution amplitudes are of minor importance, about 1%, and can be neglected safely. It is not unexpected that the main uncertainty comes from the parameter ω_0 (or λ_B), which determines the shapes of the two-particle and three-particle light-cone distribution amplitudes of the B meson. From Fig. 1, we can see that the uncertainty of the parameter λ_B almost saturates the total uncertainties, it is of great importance to refine this parameter. In this Letter, we take the value from the QCD sum rules in Ref. [32], where the B-meson light-cone distribution amplitude ϕ_+ is parameterized by the matrix element of the bilocal operator at imaginary light-cone separation.

In the region of small ω, the exponential (Gaussian) form of distribution amplitude $\phi_+(\omega)$ is numerically close to the BIK DA suggested in Ref. [32]. In Fig. 1, we also present the numerical results with the BIK DA for the central values of the input parameters λ_B and σ_B, the Gaussian distribution amplitude and the BIK DA lead to almost the same values.

From Table 1, we can see that the values of the $V_0(0)$ from the covariant light-front approach, ISGW2 quark model and quark–meson model differ greatly from the corresponding ones from the (light-cone) QCD sum rules, while the values from the (light-cone) QCD sum rules and perturbative QCD are consistent with each other. From Table 2, we observe that the values of the $A(0)$ from the covariant light-front approach, quark–meson model and perturbative QCD differ greatly from the corresponding ones from the (light-cone) QCD sum rules, while the values of the form-factors from the (light-cone) QCD sum rules are consistent with each other.

4. Conclusion

In this Letter, we calculate the weak form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$ and $A(q^2)$ with the B-meson light-cone QCD sum rules. The form-factors are basic parameters in studying the exclusive hadronic two-body decays $B\to AP$ and semi-leptonic decays $B\to Al\nu_l$, $B\to Al\bar l$. Our numerical values are consistent with the values from the (light-cone) QCD sum rules. The main uncertainty comes from the parameter ω_0 (or λ_B), which determines the shapes of the two-particle and three-particle light-cone distribution amplitudes of the B meson, it is of great importance to refine this parameter. However, it is a difficult work, as we cannot extract the values of the basic parameter λ_B directly from the experimental data on the semi-leptonic decays $B\to Al\nu_l$.

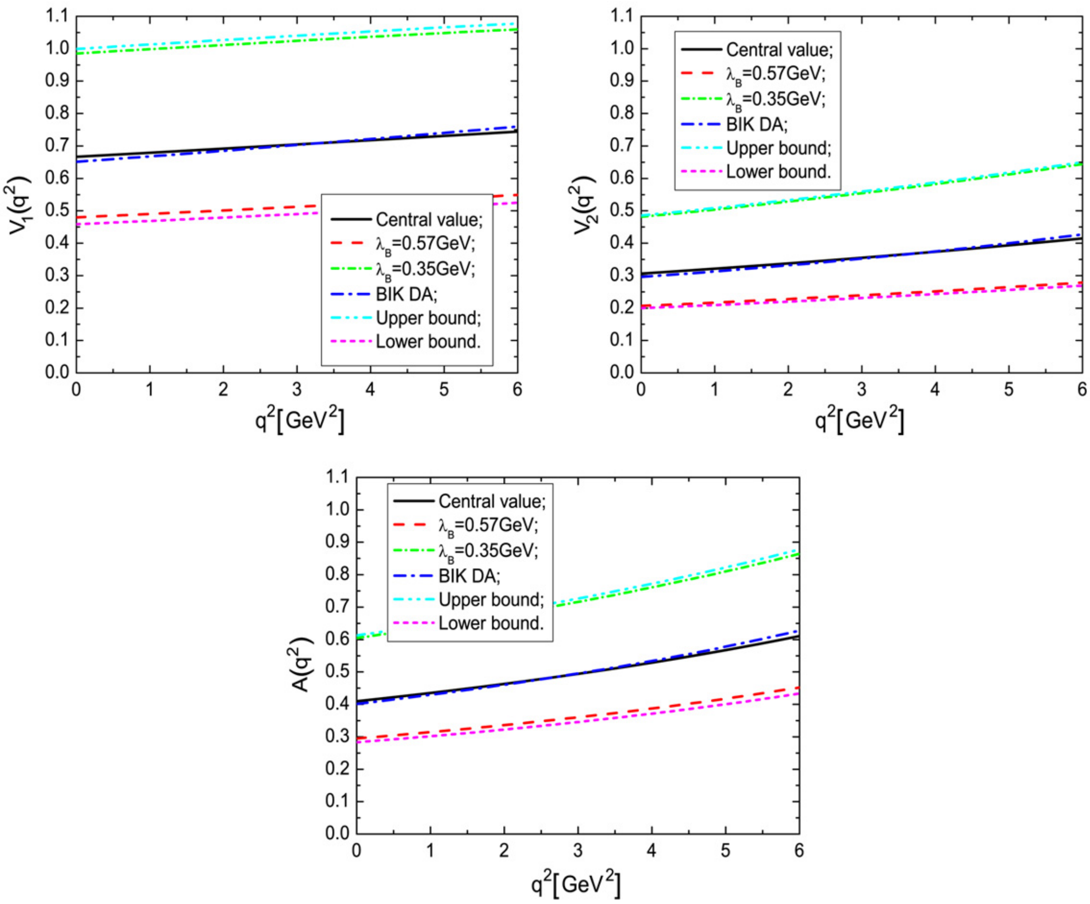

Fig. 1. The form-factors $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$ with the momentum transfer q^2.

Table 1
The form-factor $V_0(0)$ from different theoretical approaches

Theoretical approaches	$V_0(0)$
Covariant light front approach [17]	0.13
ISGW2 quark model [18]	1.01
Quark–meson model [19]	1.20
QCD sum rules [20]	0.23 ± 0.05
Perturbative QCD [21]	$0.34^{+0.07+0.08}_{-0.06-0.08}$
Light-cone sum rules [9]	0.30 ± 0.05
This work (light-cone sum rules)	$0.29^{+0.07}_{-0.06}$

Table 2
The form-factor $A(0)$ from different theoretical approaches

Theoretical approaches	$A(0)$
Covariant light front approach [17]	0.25
Quark–meson model [19]	0.09
QCD sum rules [20]	0.42 ± 0.06
Perturbative QCD [21]	$0.26^{+0.06+0.03}_{-0.05-0.03}$
This work (light-cone sum rules)	$0.41^{+0.20}_{-0.13}$

Table 3
The parameters for the fitted form-factors

	a_F	b_F
$V_1(q^2)$	−0.518	0.159
$V_2(q^2)$	−1.330	0.532
$A(q^2)$	−1.649	0.561

Acknowledgements

This work is supported by National Natural Science Foundation, Grant Number 10775051, and Program for New Century Excellent Talents in University, Grant Number NCET-07-0282.

References

[1] M. Wirbel, B. Stech, M. Bauer, Z. Phys. C 29 (1985) 637.
[2] M. Bauer, B. Stech, M. Wirbel, Z. Phys. C 34 (1987) 103.
[3] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B 606 (2001) 245.
[4] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B 591 (2000) 313.
[5] Y.Y. Keum, H.N. Li, A.I. Sanda, Phys. Rev. D 63 (2001) 054008.
[6] A. Ali, P. Ball, L.T. Handoko, G. Hiller, Phys. Rev. D 61 (2000) 074024.
[7] A. Ali, G. Kramer, C.D. Lu, Phys. Rev. D 58 (1998) 094009.
[8] K.C. Yang, Phys. Rev. D 76 (2007) 094002.
[9] H.Y. Cheng, K.C. Yang, Phys. Rev. D 76 (2007) 114020.
[10] V. Laporta, G. Nardulli, T.N. Pham, Phys. Rev. D 74 (2006) 054035.
[11] C.H. Chen, C.Q. Geng, Y.K. Hsiao, Z.T. Wei, Phys. Rev. D 72 (2005) 054011.
[12] G. Calderon, J.H. Munoz, C.E. Vera, Phys. Rev. D 76 (2007) 094019.
[13] P. Ball, R. Zwicky, Phys. Rev. D 71 (2005) 014029.
[14] P. Ball, R. Zwicky, Phys. Rev. D 71 (2005) 014015.
[15] A. Khodjamirian, R. Ruckl, S. Weinzierl, C.W. Winhart, O.I. Yakovlev, Phys. Rev. D 62 (2000) 114002.
[16] Z.G. Wang, M.Z. Zhou, T. Huang, Phys. Rev. D 67 (2003) 094006.
[17] H.Y. Cheng, C.K. Chua, C.W. Hwang, Phys. Rev. D 69 (2004) 074025.
[18] D. Scora, N. Isgur, Phys. Rev. D 52 (1995) 2783.
[19] A. Deandrea, R. Gatto, G. Nardulli, A.D. Polosa, Phys. Rev. D 59 (1999) 074012.
[20] T.M. Aliev, M. Savci, Phys. Lett. B 456 (1999) 256.
[21] W. Wang, R.H. Li, C.D. Lu, arXiv: 0711.0432.
[22] B. Aubert, et al., Phys. Rev. Lett. 97 (2006) 051802.
[23] K. Abe, et al., arXiv: 0706.3279.
[24] B. Aubert, et al., Phys. Rev. Lett. 98 (2007) 181803.
[25] B. Aubert, et al., Phys. Rev. Lett. 99 (2007) 261801.
[26] B. Aubert, et al., Phys. Rev. Lett. 100 (2008) 051803.
[27] A. Khodjamirian, T. Mannel, N. Offen, Phys. Rev. D 75 (2007) 054013.
[28] H. Kawamura, J. Kodaira, C.F. Qiao, K. Tanaka, Phys. Lett. B 523 (2001) 111.
[29] A.G. Grozin, M. Neubert, Phys. Rev. D 55 (1997) 272.
[30] H. Kawamura, J. Kodaira, C.F. Qiao, K. Tanaka, Mod. Phys. Lett. A 18 (2003) 799.
[31] B.O. Lange, M. Neubert, Phys. Rev. Lett. 91 (2003) 102001.
[32] V.M. Braun, D.Yu. Ivanov, G.P. Korchemsky, Phys. Rev. D 69 (2004) 034014.
[33] A.G. Grozin, Int. J. Mod. Phys. A 20 (2005) 7451.
[34] A. Khodjamirian, T. Mannel, N. Offen, Phys. Lett. B 620 (2005) 52.
[35] S.J. Lee, M. Neubert, Phys. Rev. D 72 (2005) 094028.
[36] K.C. Yang, Nucl. Phys. B 776 (2007) 187.
[37] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 385.
[38] L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rep. 127 (1985) 1.
[39] V.M. Belyaev, V.M. Braun, A. Khodjamirian, R. Rückl, Phys. Rev. D 51 (1995) 6177.
[40] H.n. Li, H.S. Liao, Phys. Rev. D 70 (2004) 074030.
[41] T. Huang, C.F. Qiao, X.G. Wu, Phys. Rev. D 73 (2006) 074004.
[42] P. Colangelo, A. Khodjamirian, hep-ph/0010175.
[43] Z.G. Wang, W.M. Yang, S.L. Wan, Nucl. Phys. A 744 (2004) 156.